# Improved Models for Multivariate Metocean Extremes (IMEX)

Supergen Offshore Renewable Energy

Flexible Fund Call 2


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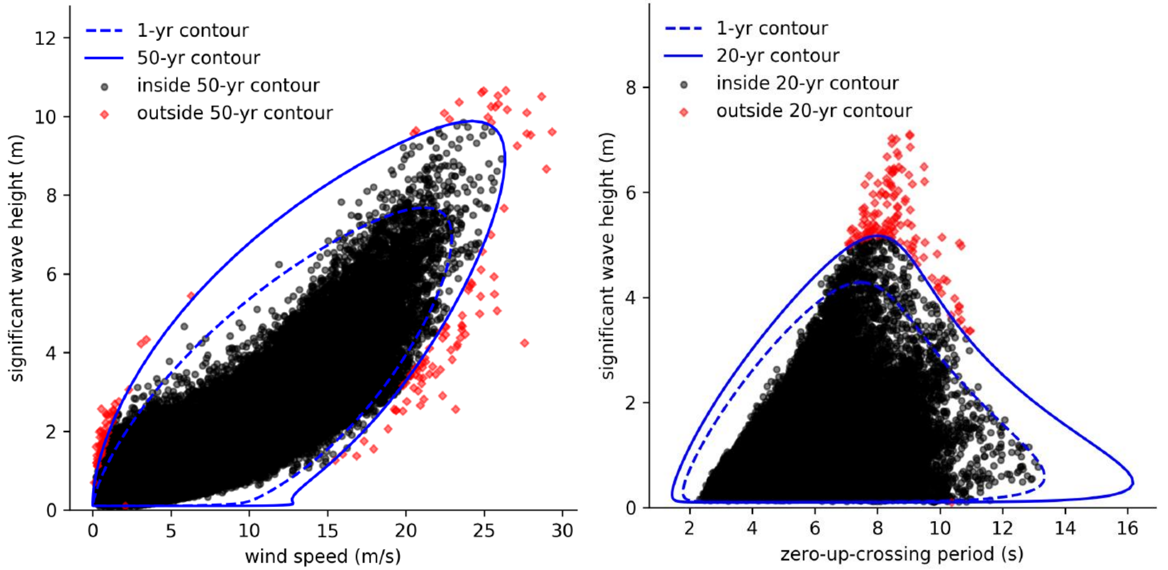
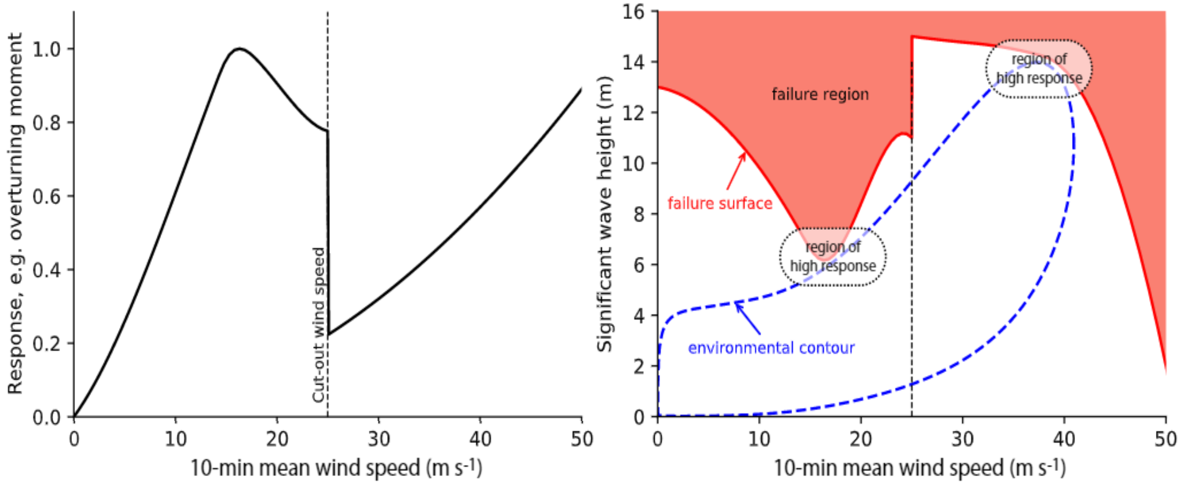
MARINE POWER SYSTEMS

# Research challenge

- Design of ORE structures requires estimates of **<u>joint extremes</u>** of winds, waves and tides
- Current design standards recommend models that make strong assumptions about form of joint distribution
- Recent benchmarking exercise demonstrated large uncertainties in current methods [1,2]
- **<u>Reduced uncertainty</u>** in environmental conditions leads to **<u>more efficient and reliable designs</u>**



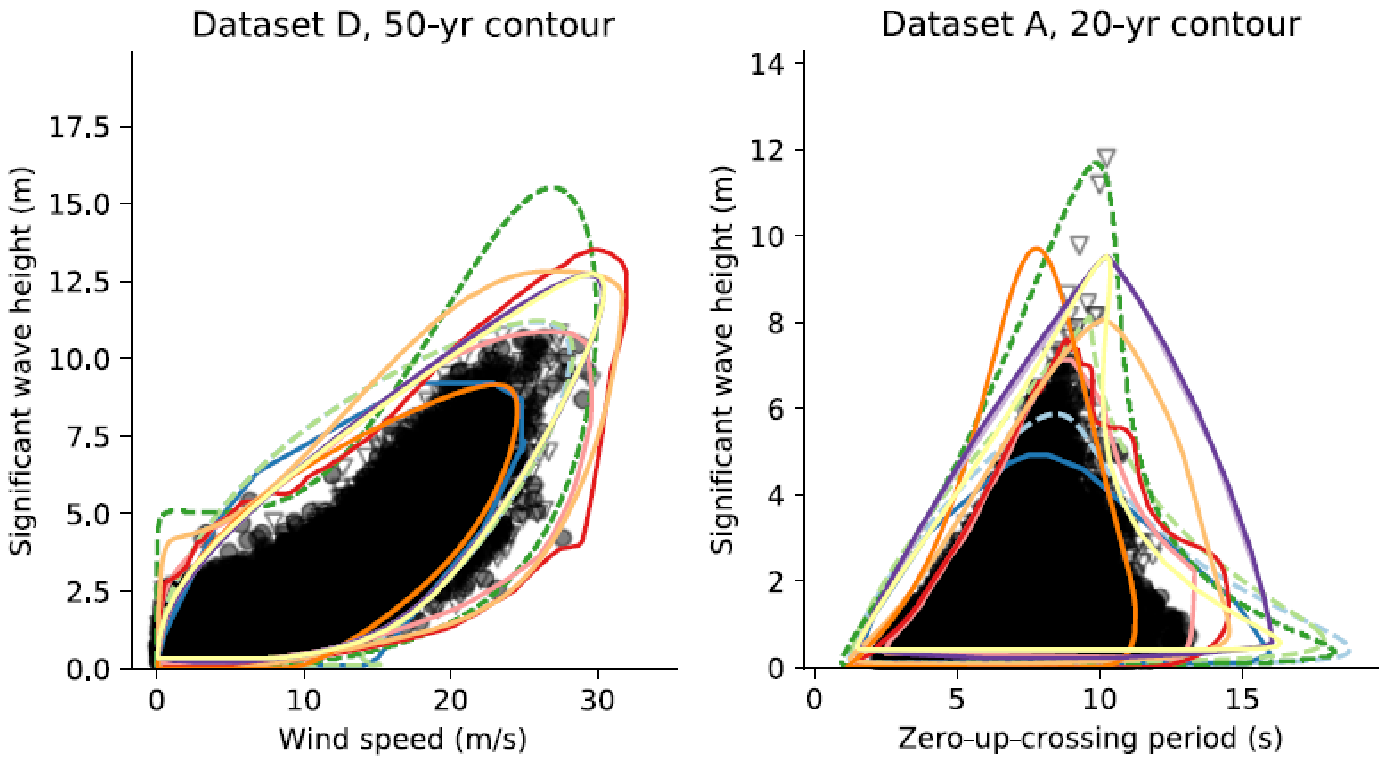

Source: [3]




Source: [4]

# Limitations with existing methods

- Based on ad hoc assumptions about marginal distributions and dependence structures
  - Not based on any physical or mathematical principle
- Often poor fits empirically
- Fitting to all observations
  - Does not ensure a good fit in the tail
  - Neglects serial correlation – leads to positive bias [5]
- Statistical models can lead to errors in extreme response of the order of ±50% [2]




Haselsteiner et al. (2021). "A benchmarking exercise for environmental contours" Ocean Engineering

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# IMEX objectives

- Develop new models for multivariate extremes
  - Based on justifiable **mathematical principles**
  - **Flexible** enough to represent observed datasets
  - **Simple** enough for routine engineering use

- Integrate models into **open-source software**

# The SPAR model

- Semi-Parametric Angular-Radial (SPAR) model [6]
- Write joint density function of $(X, Y)$ in polar coordinates $(R, \Theta)$

$$f_{R,\Theta}(r, \theta) = r.\, f_{X,Y}(x, y)$$

- Write polar density function in conditional form

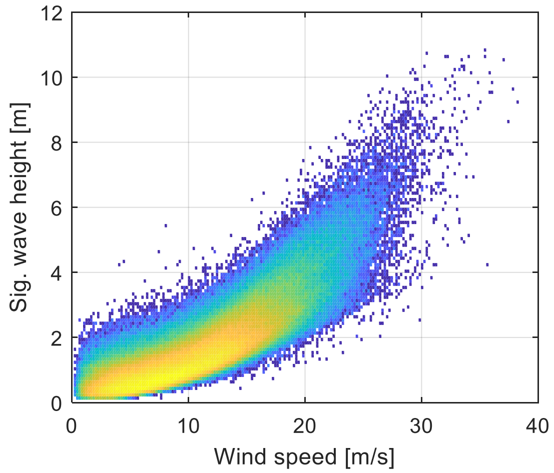
$$f_{R,\Theta}(r, \theta) = f_{\Theta}(\theta) f_{R|\Theta}(r|\theta)$$

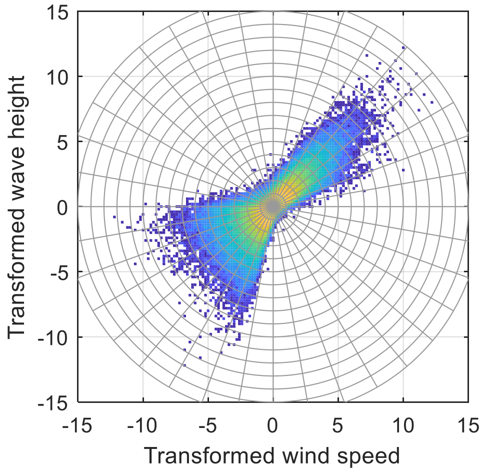
- Conditional distribution is univariate and therefore we can assume the tail converges to a generalised Pareto (GP) distribution
- SPAR model for bivariate extremes is

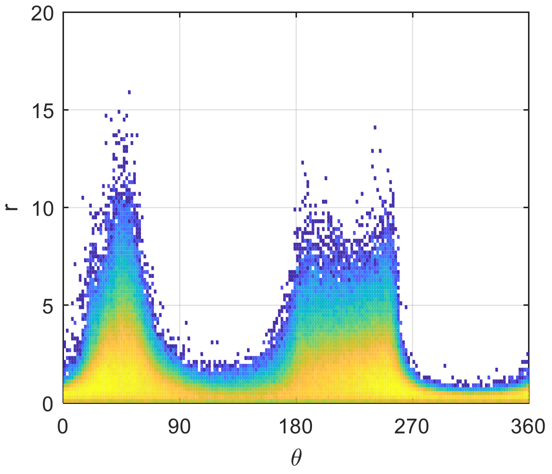
$$f_{R,\Theta}(r, \theta) = f_{\Theta}(\theta)\zeta(\theta) f_{GP}\left(r|u(\theta), \sigma(\theta), \xi(\theta)\right), \qquad r > u(\theta)$$

  - $\zeta(\theta) = \Pr\left(r > u(\theta)\right)$, assumed constant
  - $f_{GP}$ is GP density function with threshold, scale and shape, $(u, \sigma, \xi)$, conditional on angle $\theta$
- Extendable to higher dimensions in obvious way

# Example: Joint distribution of wind speed and wave height



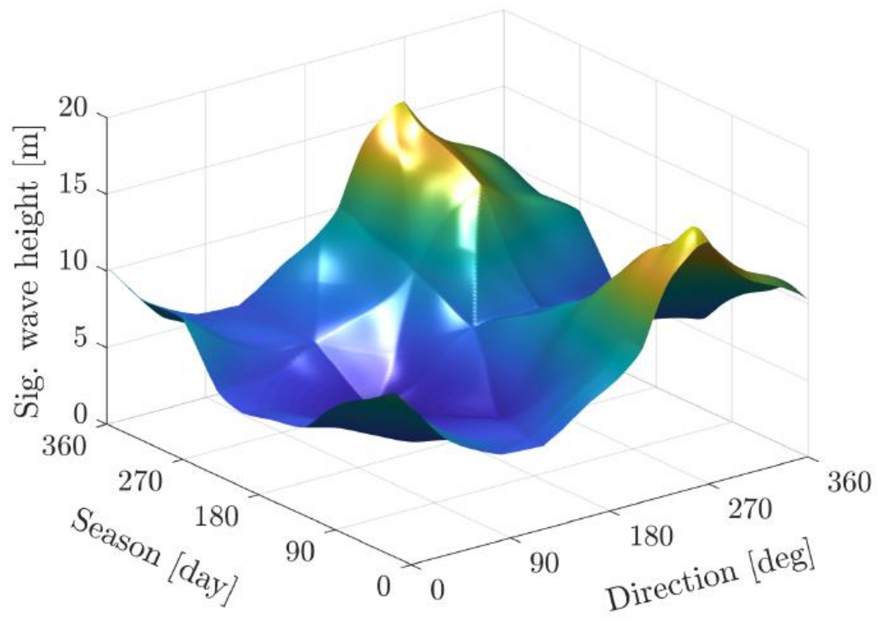




Univariate extremes with covariate dependence

# SPAR model properties

- Reframes multivariate extremes as an intuitive extension of univariate theory, with angular dependence
- Extends traditional definition of extreme as “large in one or more variables” to “locally large radial component”
- More flexible than existing methods for multivariate extremes:
  - Can represent distributions both asymptotically independent and dependent distributions
- Inference is standard univariate problem with covariate dependence [7]
- Method applicable in higher dimensions




Open-source software [7]

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# Summary

- IMEX project has delivered new models for multivariate extremes
  - Based on justifiable mathematical principles
  - More flexible than existing models

- Open source code provided at [https://github.com/edmackay/PPL-model](https://github.com/edmackay/PPL-model)

- Method is applicable for wide range of multivariate extremes problems

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# References


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