Supergen Offshore Renewable Energy

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Wave energy extraction from a floating flexible circular plate

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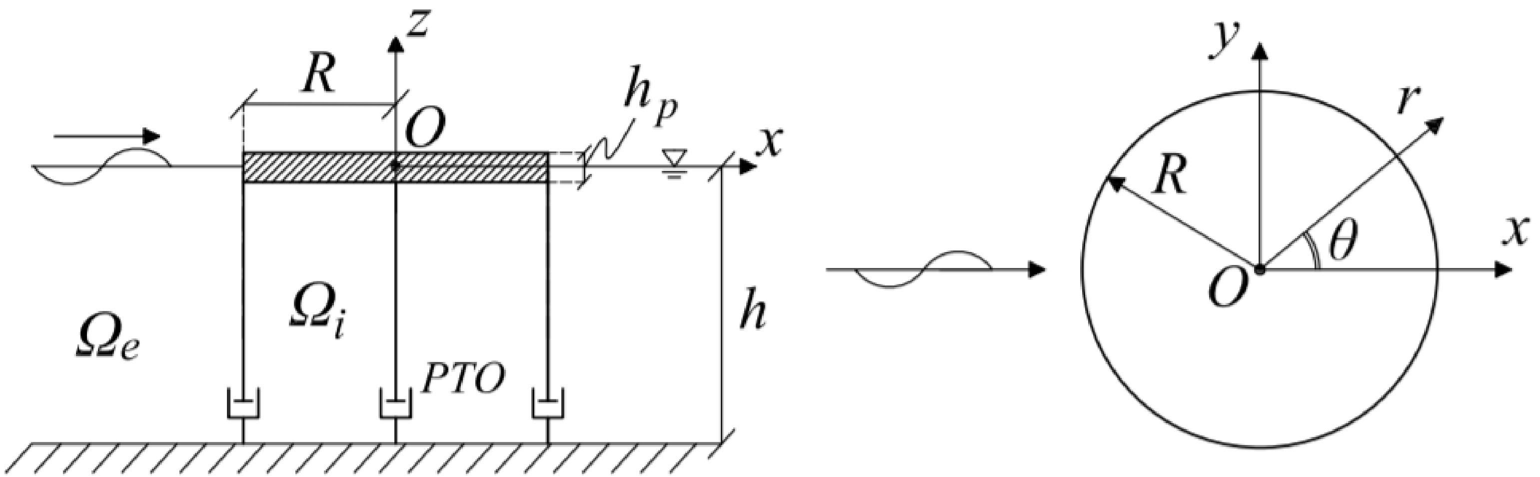
Today you will see…

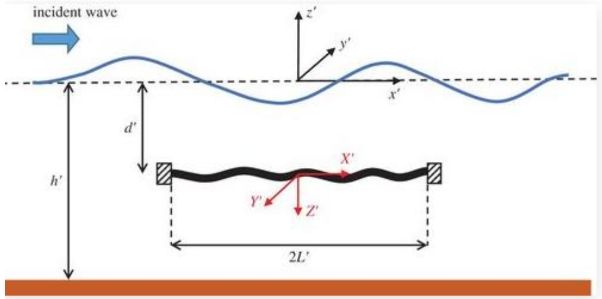
View from above and horizontal cross-section of the flexible circular WEC



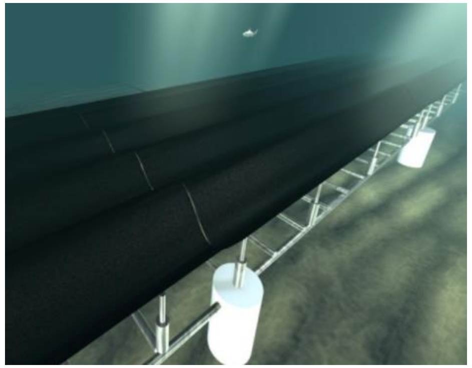
… how the properties of a floating elastic disk can be optimized to maximize wave energy extraction

Motivation of the Study

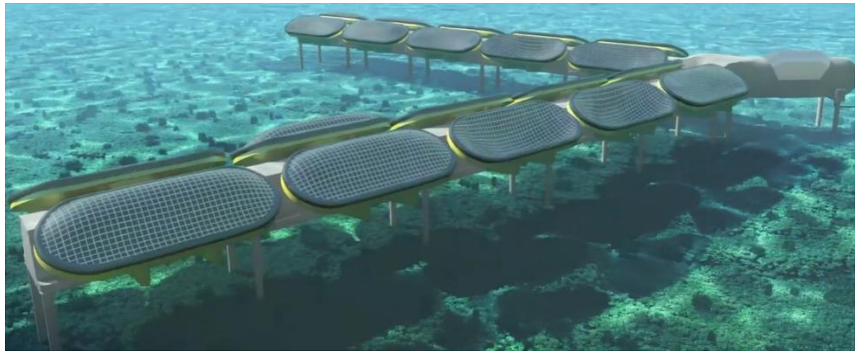
- One way to attract **funds and confidence** in industry is to consider light and flexible materials instead of bulky metallic components;
- Further **analytical/experimental work is needed** to understand the global hydrodynamic behaviour of flexible WECs;



E. Renzi. Hydroelectromechanical modelling of a piezoelectric wave energy converter (2016)

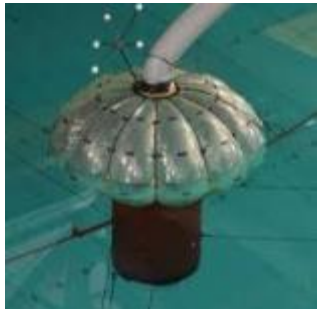


Wave Carpet



Bombora

Anaconda



Plymouth Air Bag

- In this study we propose a novel mathematical model of wave energy extraction by means of a **flexible floater**

Mathematical Model

Laplace equation in the fluid domain

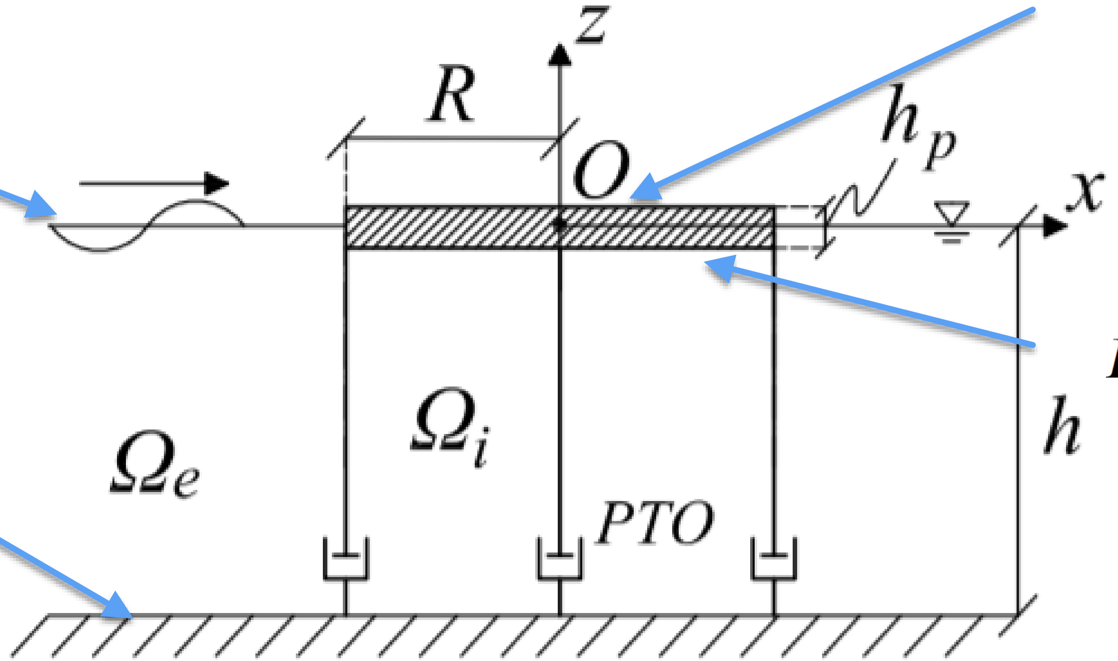
$$\nabla^2 \Phi = 0$$

Mixed b.c. on the free surface

$$\Phi_{tt} + g\Phi_z = 0$$

Kinematic b.c. on the plate wetted surface

$$\Phi_z = W_t, \quad z = 0, \; r \in [0, R]$$



Dynamic equation

$$D\nabla^4 W = q - \rho_p h_p W_{tt}, \quad r \in [0, R]$$

No-flux b.c. at the seabed

$$\Phi_z = 0, \quad z = -h,$$

Harmonic and Modal Expansion

Harmonic expansion in terms of frequency ω

$$\{\Phi, \zeta, W\} = \mathrm{Re}\left\{(\phi, \eta, w)e^{-\mathrm{i}\omega t}\right\}$$

$$\begin{cases} \nabla^2\phi = 0, & \text{in } \Omega, \\ \phi_z = -\mathrm{i}\omega\eta, & z = 0,\ r > R, \\ \phi_z = \dfrac{\omega^2}{g}\phi, & z = 0,\ r > R, \\ \phi_z = -\mathrm{i}\omega w, & z = 0,\ r \in [0, R], \\ \phi_z = 0, & z = -h, \end{cases}$$

Modal expansion into symmetric and antisymmetric modes (Newman 1994)

$$w = \zeta_h w_h + \zeta_p w_p + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \zeta_{mn} w_{mn}$$

where ζ represents the complex amplitude of each modal shape w, h denotes heave, p pitch, whereas mn denotes the bending elastic modes

Diffraction velocity potential

For $r>R$

$$\phi_D^{(e)} = -\frac{\mathrm{i}Ag}{\omega}\sum_{n=0}^{\infty}\cos n\theta\left\{\frac{\cosh k_0(h+z)}{\cosh k_0 h}\left[\epsilon_n \mathrm{i}^n J_n(k_0 r) + \mathcal{A}_{0n}^D \frac{H_n^{(1)}(k_0 r)}{H_n^{(1)\prime}(k_0 r)\big|_{r=R}}\right] + \sum_{l=1}^{\infty}\mathcal{A}_{ln}^D \frac{K_n(\overline{k_l} r)\cos\overline{k_l}(h+z)}{K_n'(\overline{k_l} r)\big|_{r=R}\cos\overline{k_l} h}\right\}$$

For $r<R$

$$\phi_D^{(i)} = -\frac{\mathrm{i}Ag}{\omega}\sum_{n=0}^{\infty}\cos n\theta\left\{\mathcal{B}_{0n}^D\left(\frac{r}{R}\right)^n + \sum_{l=1}^{\infty}\mathcal{B}_{ln}^D\frac{I_n(\mu_l r)\cos\mu_l(h+z)}{I_n'(\mu_l r)\big|_{r=R}\cos\mu_l h}\right\}, \quad \mu_l = \frac{l\pi}{h},$$

- k_0 represents the propagating wave, $\overline{k_l}$ represents the evanescent wave, $H_n^{(1)}$ is the Hankel function of first kind and order n, K_n and I_n are the modified Bessel function of order n, whereas A_{ln} and B_{ln} are unknown complex constants.
- Substituting in the matching conditions $\phi^{(i)} = \phi^{(e)}$; $\phi_r^{(i)} = \phi_r^{(e)}$ and integrating over $z \in [-h, 0]$, yields an inhomogeneous linear system in the complex constants A_{ln} , B_{ln} which can be solved numerically.

Radiation velocity potential

General solution for pitch, heave or bending mode α for $r>R$

$$\phi_\alpha^{(e)} = \sum_{n=0}^{\infty} \cos n\theta \left\{ A_{0n}^{\alpha} \frac{H_n^{(1)}(k_0 r)\cosh k_0(h+z)}{H_n^{(1)\prime}(k_0 r)\big|_{r=R} \cosh k_0 h} + \sum_{l=1}^{\infty} A_{ln}^{\alpha} \frac{K_n(\overline{k_l} r)\cos \overline{k_l}(h+z)}{K_n'(\overline{k_l} r)\big|_{r=R} \cos \overline{k_l} h} \right\}$$

The radiation potential solution in $r<R$ is given by the homogeneous part $\phi_{\alpha h}^{(i)}$ and a particular solution that accounts for the plate vibration in $z = 0$. The homogeneous component reads

$$\phi_{\alpha h}^{(i)} = \sum_{n=0}^{\infty} \cos n\theta \left\{ B_{0n}^{\alpha} \left(\frac{r}{R}\right)^n + \sum_{l=1}^{\infty} B_{ln}^{\alpha} \frac{I_n(\mu_l r)\cos \mu_l (h+z)}{I_n'(\mu_l r)\big|_{r=R} \cos \mu_l h} \right\}, \quad \mu_l = \frac{l\pi}{h},$$

Radiation velocity potential

The particular solutions for the rigid heave and pitch mode are given by

$$\tilde{\phi}_h = -\frac{i\omega}{2h}\left[z^2 + 2hz - \frac{r^2}{2}\right] \qquad \tilde{\phi}_p = -\frac{i\omega r\cos\theta}{8h}\left[4z^2 + 8hz - r^2\right]$$

whereas the particular solution for each bending elastic mode is given by

$$\tilde{\phi}_{mn} = -i\omega R\frac{\cos n\theta}{\lambda_{mn}}\left\{\frac{\cosh\frac{\lambda_{mn}(h+z)}{R}J_n\left(\frac{\lambda_{mn}r}{R}\right)}{\sinh\frac{\lambda_{mn}h}{R}} + \frac{\cos\frac{\lambda_{mn}(h+z)}{R}I_n\left(\frac{\lambda_{mn}r}{R}\right)}{\sin\frac{\lambda_{mn}h}{R}}T_{mn}\right\}, \quad n = 0, 1, \ldots$$

As before, by matching the velocity potentials in $r=R$ yields a sequence of linear systems in the unknowns $A_{ln}^{\alpha} = B_{ln}^{\alpha}$ forced by the particular solutions $\widetilde{\phi_{\alpha}}$

Plate Response

Given the effect of all the external forces, the dynamic equation can be expanded as

$$D\nabla^4 W + \rho_p h_p W_{tt} + \rho g W + \left[\sum_{i=1}^{I} \frac{1}{r}\delta\left(r - r_i\right)\delta\left(\theta - \theta_i\right) + \frac{1}{2\pi r}\delta\left(r\right)\right] \nu_{PTO} W_t + \rho \Phi_t = 0$$

where ρ is the fluid density. The third term represents the hydrostatic pressure, the fourth term denotes the effects of localised forces due to the PTO systems, δ is the Dirac delta function and the last term represents the dynamic pressure exerted by the wave field.

The complex modal amplitudes can be found by multiplying by each of the modal shape functions w and then integrating with respect the plate wetted surface.

$$\mathbf{M}\{\zeta\} = \{F\}$$

the continuous floating plate is equivalent to a system of linear coupled forced harmonic oscillators.

Generated power

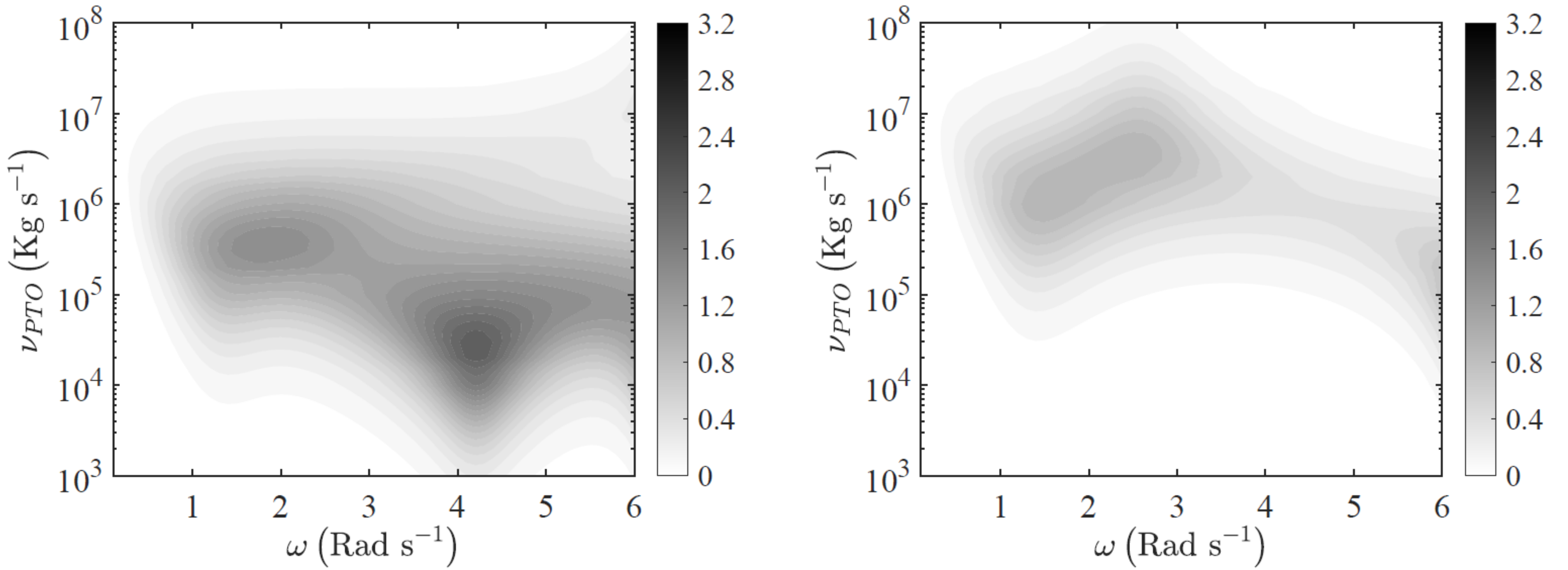
Once the response of the system is found, the average generated power by the plate and the system efficiency is simply

$$P = \frac{1}{2}\nu_{PTO}\omega^2 \sum_{i=1}^{I} \left| w\left(r_i, \theta_i\right) \right|^2 \Longrightarrow C_W = \frac{k_0 P}{E C_g}, \quad E = \frac{1}{2}\rho g A^2,$$

Where C_W represents the efficiency, w is the amplitude of plate oscillations corresponding to the PTO location (r_i, θ_i) and E is the total energy

Effects of the PTO distribution

Let us consider the following parameters: $A = 1\ m$, $h = 10\ m$, $h_p = 1\ m$, $\upsilon = 0.3$, $E = 0.1\ GPa$ *and* $R = 10m$

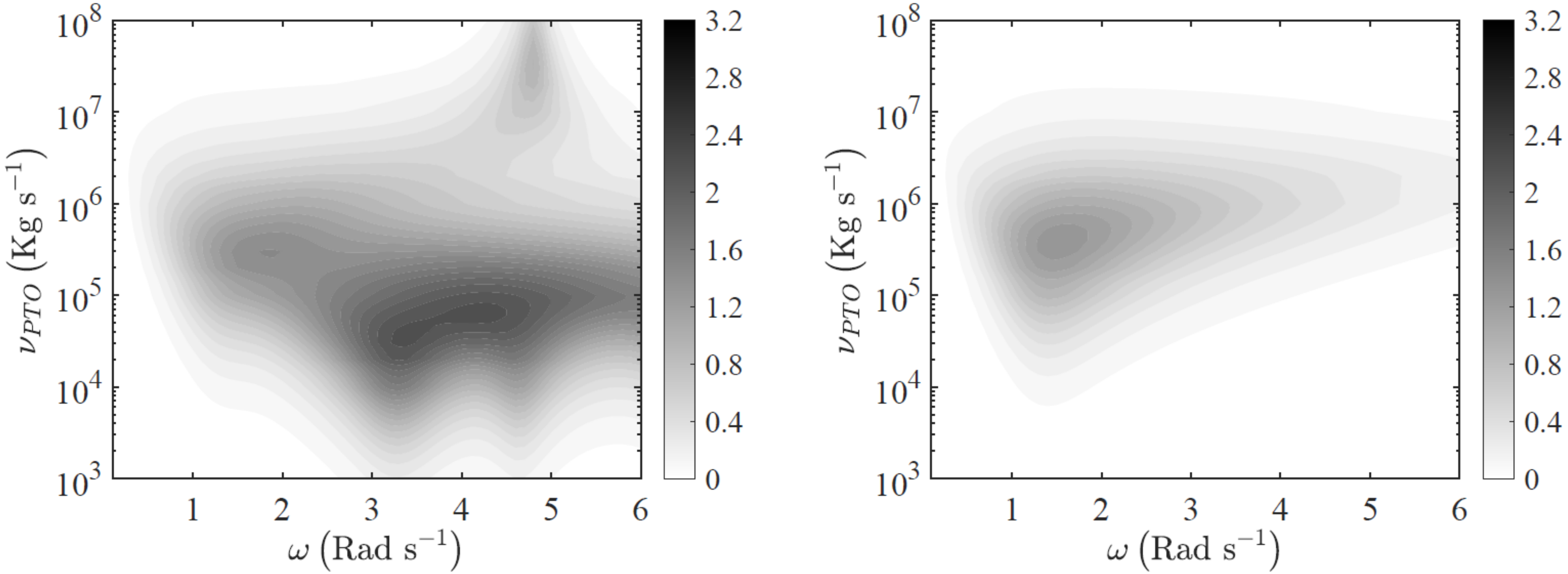


Behaviour of C_W versus frequency of the incident waves ω and PTO-Coefficient. (a) 5 PTO devices located in $r = 0$ and $r_i = R$, $\theta_i = [0, \frac{\pi}{2}, \pi, 3\pi/2]$ (b) Single PTO device located at the center of the plate.

When the number of the PTOs increases, the bandwidth of C_W increases and the system becomes more efficient. From a theoretical point of view, this distribution maximises the radiated waves in the direction opposite the incident waves

Effects of plate flexural rigidity

Young's modulus $E = 0.05\ GPa$ and an idealized rigid plate. The 5 PTO devices are located in $r = 0$ and $r_i = R$, $\theta_i = [0, \frac{\pi}{2}, \pi, 3\pi/2]$ whereas the plate radius is $R = 10\ m$.

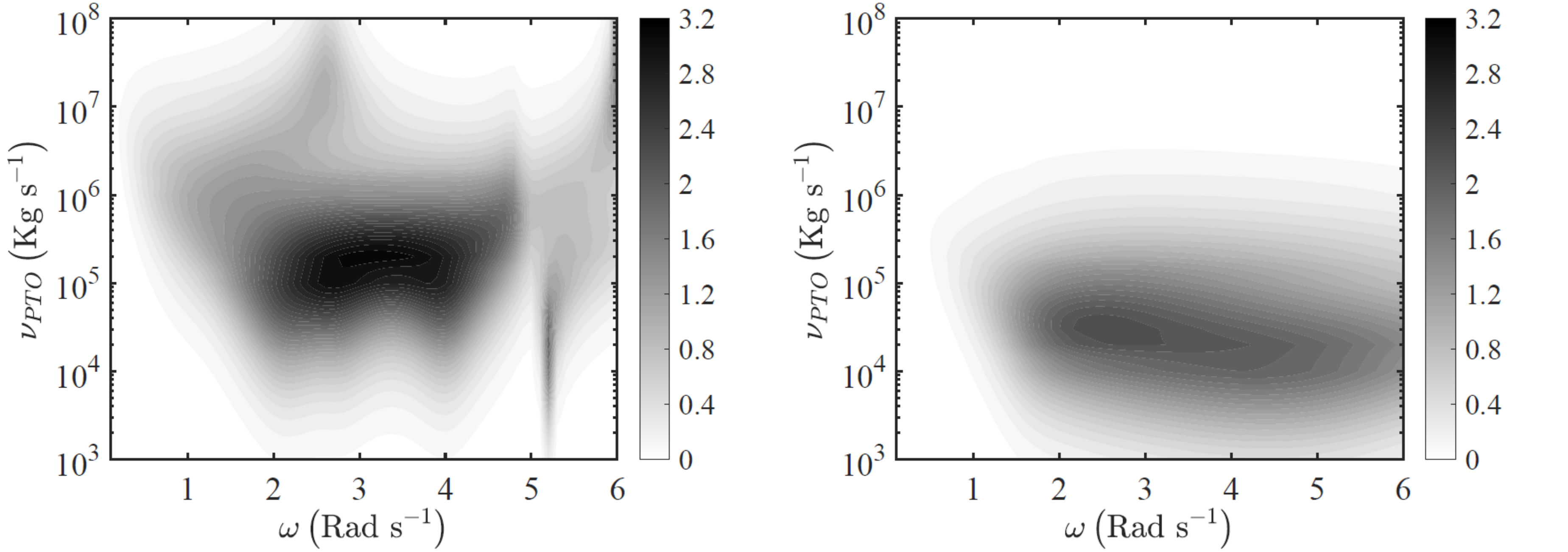


Behaviour of C_W versus frequency of the incident waves ω and PTO-Coefficient. (a) Flexible softened plate (b) Rigid plate

When the flexural rigidity of the plate decreases, the efficiency of the system increases significantly.

Effects of plate dimensions

Consider two different values of plate radius *R* =[15; 5] *m*, fixed Young's modulus *E* = 0.1 *GPa* and fixed PTO distribution. The 5 PTO devices are in *r* = 0 and $r_i = R$, $\theta_i = [0, \frac{\pi}{2}, \pi, 3\pi/2]$ whereas the plate radius is *R* = 10 *m*.



Behaviour of C_W versus frequency of the incident waves ω and PTO-Coefficient. (a) R=15m (b) R=5 m

For *R*=15 *m* the efficiency bandwidth and C_W increase because of increased number of eigenfrequencies in the range of interest.

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THANK YOU

Any questions?

References

- S. Michele, S. Zheng and D. Greaves. 2022 Wave energy extraction from a floating flexible circular plate. Ocean Engineering. Accepted.
- S. Michele, F. Buriani, E. Renzi, M. van Rooij, B. Jayawardhana, A. Vakis. 2020. Wave energy extraction by flexible floaters. Energies.